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Casimir forces for moving boundaries with Robin conditions

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Abstract

We consider a massless scalar field in 1+1 dimensions that satisfies a Robin boundary condition at a non-relativistic moving boundary. Using the perturbative approach introduced by Ford and Vilenkin, we compute the total force on the moving boundary. In contrast to what happens for the Dirichlet and Neumann boundary conditions, in addition to a dissipative part, the force acquires also a dispersive one. Further, we also show that with an appropriate choice for the mechanical frequency of the moving boundary it is possible to turn off the vacuum dissipation almost completely.

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1. Introduction and physical motivations

The interaction between a physical system and a material plate (or cavity in general) in its surroundings has a long history. In 1948, Casimir and Polder [1] computed for the first time the retarded interaction energy between a neutral but polarizable atom and a perfectly conducting wall. In this same year, Casimir [2] predicted the attraction between two neutral parallel conducting plates due to the shift caused by the plates in the energy of the radiation field in the vacuum state. Casimir's result may be considered the first problem worked out in detail of the so-called cavity QED. Since then, a lot of work has been done on the Casimir effect; see for instance the reviews [3–8] and references therein (for other phenomena of cavity QED, see [9, 10]).

However, the interaction between a quantum field and a material plate is quite complicated. Hence, as a first approximation, it is common to simulate this interaction by imposing an idealized boundary condition on the field . The most familiar conditions are Dirichlet and Neumann ones. A less familiar, but no less important condition is the so-called Robin boundary condition, defined for a scalar field by

$$\phi\Big|_{\partial\mathcal{R}} = \beta\frac{\partial\phi}{\partial n}\Big|_{\partial\mathcal{R}}, \tag{1}$$

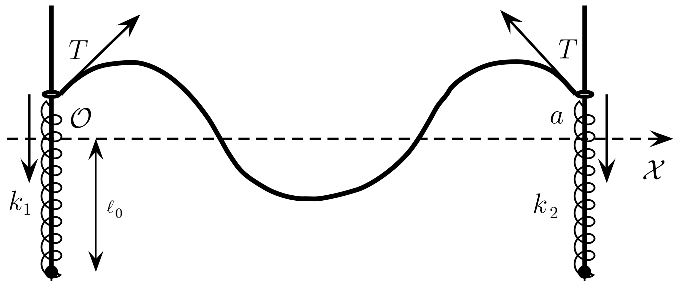

Figure 1. Elastic supports at $x = 0$ and $x = a$ give rise to Robin BC.

where $\partial\mathcal{R}$ is the boundary of the system under study, $\frac{\partial\phi}{\partial n}$ means $\hat{\mathbf{n}} \cdot \nabla\phi$, with $\hat{\mathbf{n}}$ being a unitary vector normal to the boundary and β is a parameter with dimension of length that can assume any value in the interval $[0, \infty)$. Robin BC have the nice property of interpolating continuously Dirichlet and Neumann ones. From (1), we immediately see that for $\beta = 0$ we have Dirichlet BC and for $\beta \to \infty$ we have Neumann BC.

In this work, we discuss some consequences of using Robin BC in the context of the dynamical Casimir effect. However, before starting our calculations, we shall make a few comments about this kind of BC. Robin BC already appear in a natural way in classical physics. For instance, when we solve problems in classical electromagnetism in the presence of spherical conducting shells the radial functions satisfy Robin BC with particular values of parameter β. Another nice example, still in the context of classical physics, is the problem of a vibrating string subjected to a tension T with two massless rings at its ends which may slide without friction along vertical rods and are coupled to springs of constants κ_1 and κ_2, respectively, as indicated in figure 1.

Assuming small inclinations $\left(\left|\frac{\partial y}{\partial x}\right| \ll 1\right)$, application of Newton's second law to both massless rings gives

$$y\Big|_{x=0} = \frac{T}{\kappa_1}\frac{\partial y}{\partial x}\Big|_{x=0} \qquad \text{and} \qquad y\Big|_{x=a} = -\frac{T}{\kappa_2}\frac{\partial y}{\partial x}\Big|_{x=a}. \tag{2}$$

The fact that Robin BC simulate an elastic support at the boundary has been pointed out in the literature [11]. Though the reflection at a fixed boundary where the wave satisfies a Robin BC is complete, there is some kind of time delay caused by a bulk/boundary dynamics. In other words, the reflection coefficient can be written as $R = \mathrm{e}^{\mathrm{i}\phi(k)}$ (note that $|R| = 1$), where k is the wavenumber of the incident wave and hence there will be a phase shift between the incident and reflected waves. This gives a qualitative explanation for the surface terms that appear in connection with Robin BC in quantum field theory [12–15]. Total energy (string plus surface terms) is conserved, but there is a 'bulk/boundary' exchange, so that the energy of the string itself is not conserved:

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_0^a \left[\frac{1}{2}\mu\left(\frac{\partial y}{\partial t}\right)^2 + \frac{1}{2}T\left(\frac{\partial y}{\partial x}\right)^2\right]\mathrm{d}x = -\left\{\kappa_1 y(0,t)\frac{\partial y}{\partial t}(0,t) + \kappa_2 y(a,t)\frac{\partial y}{\partial t}(a,t)\right\}.$$

Robin BC are also useful for phenomenological models that describe penetrable surfaces [19]. In fact, for some particular cases, these conditions can simulate the plasma model for real metals. It is not difficult to show that for frequencies much smaller than the plasma frequency, $\omega \ll \omega_P$, a small value of β plays the role of $1/\omega_P$ ($c = 1$). In other words, under such assumptions, β is proportional to the plasma wavelength, which is directly related to the penetration depth of the field.

Recently, Robin BC have been studied in many different contexts, namely: Bondurant and Fulling [16] discussed in detail Green's functions of the wave, heat and Schrödinger equations under Robin BC; Albuquerque and Cavalcanti [12] and Albuquerque [24] analysed the one-loop renormalization of a $\lambda\phi^4$ theory under these conditions; Minces and Rivelles used them in the context of AdS/CFT correspondence; Solodukihn [20] studied upper bounds for the ratio between entropy and energy of systems constrained by Robin BC; heat kernel coefficients were studied by Bordag *et al* [22], Fulling [13] and Dowker [23]; and a very detailed calculation of the static Casimir effect with Robin BC was made by Romeo and Saharian [15]. It is worth mentioning that Robin BC may give rise to restoring Casimir forces between two parallel plates, once parameters β at each plate are appropriately chosen.

However, since the pioneering paper by Moore [25] on radiation reaction forces on moving boundaries, Robin BC have never been considered explicitly in the context of the dynamical Casimir effect (as far as the authors know). It is our purpose here to make this kind of calculation in a simple model, namely, we shall consider a massless scalar field ϕ in 1+1 dimensions subjected to a Robin BC at one non-relativistic moving boundary. The main motivation is the following: it has been shown that for Dirichlet [26–28] and Neumann BC [29] the linear susceptibilities are equal and purely imaginary,

$$\chi^D(\omega) = \chi^N(\omega) = \mathrm{i}\frac{\hbar\omega^3}{6\pi c^2}. \tag{3}$$

These susceptibilities lead to purely dissipative forces on the moving boundary:

$$\delta F^D(t) = \delta F^N(t) = \frac{\hbar}{6\pi c^2}\frac{\mathrm{d}^3}{\mathrm{d}t^3}\delta q(t), \tag{4}$$

where $\delta q(t)$ is the position of the moving boundary at instant t.

For more general BC see Jaekel and Reynauld [28, 30], and for 3+1 calculations see [31]. Since Robin BC interpolates continuously Dirichlet and Neumann ones we are led to the following questions. What happens to the force for the interpolating BC? Will it still be a purely dissipative one? In what follows we shall answer these questions.

2. Casimir forces with Robin boundary conditions

Besides the assumption of a non-relativistic motion for the boundary, we shall also suppose that the boundary has a prescribed motion with a small amplitude, $\delta q(t)$ being its position at time t. Hence, we assume that

$$|\delta\dot{q}(t)| \ll c \qquad \text{and} \qquad |\delta q(t)| \ll c/\omega_0, \tag{5}$$

where ω_0 corresponds to the typical mechanical frequency. Therefore, we need to solve the wave equation for the quantum field, $\partial^2\phi(t,x) = 0$, with ϕ satisfying a Robin BC at the moving boundary, which, in the comoving frame, is written as

$$\left.\frac{\partial\phi'}{\partial x'}(t',x')\right|_{\text{Bound}} = \left.\frac{1}{\beta}\phi'(t',x')\right|_{\text{Bound}}. \tag{6}$$

The corresponding BC in the laboratory frame is given by

$$\left.\left[\frac{\partial}{\partial x} + \delta\dot{q}(t)\frac{\partial}{\partial t}\right]\phi(t,x)\right|_{x=\delta q(t)} = \left.\frac{1}{\beta}\phi(t,x)\right|_{x=\delta q(t)}, \tag{7}$$

where we neglected terms of $\mathcal{O}(\delta\dot{q}^2/c^2)$. Using the perturbative approach of Ford and Vilenkin [27] we write

$$\phi(x,t) = \phi_0(x,t) + \delta\phi(x,t), \tag{8}$$

where ϕ_0 is the solution with a static boundary at $x = 0$, which is given by

$$\phi_0(t,x) = \int_0^\infty \frac{d\omega}{\sqrt{\omega(1+\omega^2\beta^2)\pi}}[\sin(\omega x) + \omega\beta\cos(\omega x)][a(\omega)\,e^{-i\omega t} + a^\dagger(\omega)\,e^{i\omega t}], \tag{9}$$

and $\delta\phi$ corresponds to the contribution generated by the movement of the boundary. This perturbation satisfies the wave equation $\partial^2\delta\phi(x,t) = 0$ with the following BC:

$$\frac{\partial\delta\phi}{\partial x}(t,0) - \frac{1}{\beta}\delta\phi(t,0) = \delta q(t)\left[\frac{1}{\beta}\frac{\partial\phi_0}{\partial x}(t,0) - \frac{\partial^2\phi_0}{\partial x^2}(t,0)\right] - \delta\dot{q}(t)\frac{\partial\phi_0}{\partial t}(t,0), \tag{10}$$

where we discarded terms of $\mathcal{O}(\delta q^2)$. The total force on the boundary is given by

$$\delta F(t) = \langle 0|T^{11}(t,\delta q^+(t)) - T^{11}(t,\delta q^-(t))|0\rangle, \tag{11}$$

where $T^{11}(t,x) = -\frac{1}{2}\{(\partial_x\phi)^2(t,x) + (\partial_t\phi)^2(t,x)\}$. Substituting $\phi = \phi_o + \delta\phi$, we get

$$\delta F(t) = -\tfrac{1}{2}\langle 0|(\{(\partial_x\phi_0)(t,\delta q^+(t)), (\partial_x\delta\phi)(t,\delta q^+(t))\}$$
$$+ \{(\partial_t\phi_0)(t,\delta q^+(t)), (\partial_t\delta\phi)(t,\delta q^+(t))\} - [\delta q^+(t) \to \delta q^-(t)])|0\rangle + \mathcal{O}(\delta\phi^2).$$

In the last equation, $\{\ldots,\ldots\}$ means anticommutator and terms involving only the non-perturbed field ϕ_0 disappear. Now, we expand around $x = 0$ and keep only first-order terms. One may also show that the total force is twice the force on each side. With these facts in mind, we get

$$\delta F(t) = -\tfrac{1}{2}\langle 0|(\{(\partial_x\phi_0)(t,0^+), (\partial_x\delta\phi)(t,0^+)\}$$
$$+ \{(\partial_t\phi_0)(t,0^+), (\partial_t\delta\phi)(t,0^+)\} - [0^+ \to 0^-])|0\rangle$$
$$= -\langle 0|\{(\partial_x\phi_0)(t,0^+), (\partial_x\delta\phi)(t,0^+)\} + \{(\partial_t\phi_0)(t,0^+), (\partial_t\delta\phi)(t,0^+)\}|0\rangle.$$

Denoting by $\delta\mathcal{F}(\omega)$, $\delta\Phi(\omega,x)$ and $\delta Q(\omega)$ the time Fourier transforms of $\delta F(t)$, $\delta\phi(t,x)$ and $\delta q(t)$, respectively, it is straightforward to show that

$$\delta\mathcal{F}(\omega) = -\int\frac{d\omega'}{2\pi}(\langle 0|\{\partial_x\Phi_0(\omega-\omega',0), \partial_x\delta\Phi(\omega',0)\}$$
$$-(\omega-\omega')\omega'\{\Phi_0(\omega-\omega',0), \delta\Phi(\omega',0)\})|0\rangle.$$

Hence, we must solve the equation $\left(\partial_x^2 + \omega^2\right)\delta\Phi(x,\omega) = 0$ with the BC (this is condition (7) translated to the Fourier space):

$$\partial_x\delta\Phi(\omega',0) - \frac{1}{\beta}\delta\Phi(\omega',0) = \frac{1}{\beta}\int\frac{d\omega''}{2\pi}\partial_x\Phi_0(\omega'',0)\delta Q(\omega'-\omega'')$$
$$+\int\frac{d\omega''}{2\pi}\omega'\omega''\Phi_0(\omega'',0)\delta Q(\omega'-\omega'').$$

However, $\delta\Phi(\omega,x)$ satisfies a second-order differential equation, which means that we shall need an extra condition. A natural choice is to consider only the solutions for $\delta\phi(t,x)$ which describe perturbations getting away from the boundary:

$$\delta\Phi(\omega,x) = \mathrm{sgn}(x)\frac{1}{i\omega}\partial_x\delta\Phi(\omega,0)\,e^{i\omega|x|} \quad\Longrightarrow\quad \delta\Phi(\omega,0^\pm) = \pm\frac{1}{i\omega}\partial_x\delta\Phi(\omega,0). \tag{12}$$

The last equations allow us to express $\delta\Phi(\omega,0)$ and $\partial_x\delta\Phi(\omega,0)$ in terms of the static field. The resulting expressions, when substituted in $\delta\mathcal{F}(\omega)$, give

$$\delta\mathcal{F}(\omega) = \int\frac{d\omega'}{2\pi}\left(\frac{\beta\omega'}{i+\beta\omega'}\right)\int\frac{d\omega''}{2\pi}\delta Q(\omega'-\omega'')$$
$$\times\left(-\frac{1}{\beta}\langle 0|\{\partial_x\Phi_0(\omega-\omega',0), \partial_x\Phi_0(\omega'',0)\}|0\rangle\right.$$
$$\left.+\cdots+(\omega-\omega')(\omega'-\omega'')\omega''\langle 0|\{\Phi_0(\omega-\omega',0), \Phi_0(\omega'',0)\}|0\rangle\right),$$

where $\cdots$ means other (though analogous) correlators. However, all correlators in the previous expression are connected by the field equation and Robin BC to the following one: $C_0(\omega_{1,}\omega_2) = \langle 0|\{\Phi_0(\omega_1, 0), \Phi_0(\omega_2, 0)\}|0\rangle$, which involves only the non-perturbed field. A straightforward calculation leads to

$$C_0(\omega_1, \omega_2) = \frac{4\pi\beta^2}{\left(1+\omega_1^2\beta^2\right)}|\omega_1|\delta(\omega_1+\omega_2). \tag{13}$$

With the aid of this correlator, we write $\delta\mathcal{F}(\omega)$ in the form

$$\delta\mathcal{F}(\omega) =: \chi(\omega)\delta Q(\omega), \tag{14}$$

where the real and imaginary parts of the susceptibility $\chi(\omega)$ are identified as

$$\mathrm{Re}\chi(\omega) = -\frac{\omega\beta}{\pi}\int_{-\infty}^{\infty}\mathrm{d}\omega'\frac{\omega'|\omega'-\omega|[1+\beta^2\omega'(\omega'-\omega)]}{[\beta^2(\omega'-\omega)^2+1](\beta^2\omega'^2+1)} \tag{15}$$

and

$$\mathrm{Im}\chi(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty}\mathrm{d}\omega'\frac{\omega'|\omega'-\omega|\{1+2\beta^2\omega'(\omega'-\omega)+\beta^4\omega'^2(\omega'-\omega)^2\}}{[\beta^2(\omega'-\omega)^2+1](\beta^2\omega'^2+1)}. \tag{16}$$

Before we proceed, it is interesting to check some limits. Taking the limits $\beta = 0$ (Dirichlet BC) or $\beta \to \infty$ (Neumann BC) in the above expressions, we obtain

$$\mathrm{Re}\chi(\omega) \longrightarrow 0 \qquad \text{and} \qquad \mathrm{Im}\chi(\omega) \longrightarrow \frac{\mathrm{i}}{\pi}\int_{-\infty}^{+\infty}\mathrm{d}\omega'\omega'|\omega-\omega'|. \tag{17}$$

As anticipated, the susceptibility is purely imaginary in these limits. In order to perform the integration for $\mathrm{Im}\chi(\omega)$ we need a regularization prescription. In this case, a very natural way of doing that is to write the integral in the form

$$\left\{\int_{-\infty}^{+\infty}\mathrm{d}\omega'\omega'|\omega-\omega'|\right\}^{\mathrm{reg}} = \lim_{L\to\infty}\left(\int_{-L}^{0}+\int_{0}^{\omega}+\int_{\omega}^{\omega+L}\right)\mathrm{d}\omega'\omega'|\omega-\omega'|. \tag{18}$$

Note that the first and third integrals on the right-hand side of the above equation cancel out. Therefore, we are left with

$$\chi(\omega) = \mathrm{i}\frac{\hbar}{\pi}\int_0^{\omega}\mathrm{d}\omega'\,\omega'|\omega-\omega'| = \mathrm{i}\frac{\hbar\omega^3}{6\pi}, \tag{19}$$

which leads to the well-known forces already written in (4). For later convenience, it is worth emphasizing that the total work is given by

$$\int_{-\infty}^{+\infty}F(t)\delta\dot{q}(t)\,\mathrm{d}t = -\frac{1}{\pi}\int_0^{\infty}\mathrm{d}\omega\,\omega\,\mathrm{Im}\chi(\omega)|\delta Q(\omega)|^2. \tag{20}$$

Note that, for Dirichlet and Neumann BC, $\mathrm{Im}\chi(\omega) > 0$ for $\omega > 0$, so that the force is always dissipative.

Using a regularization prescription analogous to that described above in equations (15) and (16), the contributions for the integrals coming from the intervals $(-\infty, 0)$ and (ω, ∞) will cancel each other and we are left with integrals from 0 to ω. Performing the remaining integrals, we obtain

$$\mathrm{Re}\chi(\omega) = \frac{\omega}{\beta^2\pi}\frac{-\beta\omega\log(\beta^2\omega^2+1)+\beta^3\omega^3+4\beta\omega-2\tan^{-1}(\beta\omega)(\beta^2\omega^2+2)}{\beta^2\omega^2+4}$$

$$\mathrm{Im}\chi(\omega) = \frac{\omega}{6\beta^2\pi}\frac{\beta^4\omega^4+4\beta^2\omega^2-6\log(\beta^2\omega^2+1)(\beta^2\omega^2+2)+12\beta\omega\tan^{-1}(\beta\omega)}{\beta^2\omega^2+4}.$$

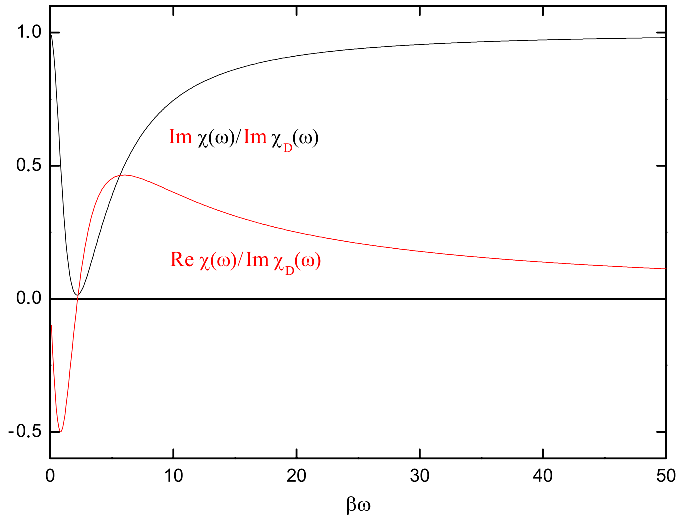

Figure 2. Real and imaginary parts of $\chi(\omega)$, conveniently normalized by $\mathrm{Im}\chi_D(\omega)$, as functions of $\beta\omega$.

Expanding the previous expressions appropriately, the first corrections to the Dirichlet and Neumann cases can be obtained.

For $\beta\omega \ll 1$, we have

$$\mathrm{Re}\chi(\omega) = -\frac{\omega^4}{6\pi}\beta + \frac{2\omega^6}{15\pi}\beta^3 + \mathcal{O}(\beta^5) \qquad \text{and} \qquad \mathrm{Im}\chi(\omega) = \frac{\omega^3}{6\pi} - \frac{\omega^5}{6\pi}\beta^2 + \mathcal{O}(\beta^4).$$

For $\beta\omega \gg 1$, we have

$$\mathrm{Re}\chi(\omega) = -\frac{\omega^2}{\pi}\frac{1}{\beta} - \frac{\omega}{\beta^2} + \mathcal{O}(\beta^{-3}) \qquad \text{and} \qquad \mathrm{Im}\chi(\omega) = \frac{\omega^3}{6\pi} - \frac{2\omega}{\pi}\frac{\log(\beta\omega)}{\beta^2} + \mathcal{O}(\beta^{-4}).$$

Hence, the total force is given by

$$\delta F(t) = \frac{1}{6\pi}\left\{\frac{\mathrm{d}^3}{\mathrm{d}t^3}\delta q(t) - \beta\frac{\mathrm{d}^4}{\mathrm{d}t^4}\delta q(t)\right\} + \mathcal{O}(\beta^2) \qquad (\beta \to 0)$$

$$\delta F(t) = \frac{1}{\pi}\left\{\frac{1}{6}\frac{\mathrm{d}^3}{\mathrm{d}t^3}\delta q(t) - \frac{1}{\beta}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\delta q(t)\right\} + \mathcal{O}(\beta^{-3}) \qquad (\beta \to \infty).$$

The behaviour of $\mathrm{Re}\chi(\omega)$ and $\mathrm{Im}\chi(\omega)$ is shown in figure 2.

3. Conclusion

In this work, we computed the total force on a non-relativistic moving boundary in 1+1 dimensions due to the vacuum fluctuations of a massless scalar field subjected to Robin BC. Dirichlet and Neumann BC correspond to particular limits of our results (particular values of β). It is worth emphasizing that when Robin BC are used the susceptibility acquires a real part. The pronounced valley in the graph of $\mathrm{Im}\chi(\omega)/\mathrm{Im}\chi_D(\omega)$ leads to a quite interesting result: if $\delta Q(\omega)$ is peaked around ω_0, equation (20) shows that, for any fixed β, there will be an appropriate choice of ω_0 such that the dissipative effects on the boundary will be almost completely eliminated. A natural sequence of this work is to compute the particle creation rate under the same circumstances as those assumed in this work. This problem is under study and the results will be published elsewhere.

Acknowledgments

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